ON THE RESOLUTION OF AN ARBITRARY DISCONTINUITY IN MAGNETOHYDRODYNAMICS

(O RASPADE PROIZVOL'NOGO RAZRYVA V MAGNITNOI GIDRODINAMIKE)

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The problem is considered of the resolution of a discontinuity in magnetohydrodynamics with a magnetic field perpendicular to the plane of the discontinuity. The parameters of the medium on both sides of the discontinuity are arbitrary. Altogether twenty different cases of resolution are possible. This paper shows which of the possible cases of resolution, can be realized as a function of the values of the initial parameters of the medium.

The resolution of an arbitrary discontinuity in gasdynamics was considered in [1-4]. The resolution of an arbitrary discontinuity in magnetohydrodynamics without limitation on the parameters of the medium and the magnetic field was considered in [5-7]. Certain special cases of the problem were investigated in [8-13].

The present paper investigates the case of resolution of an arbitrary discontinuity in magnetohydrodynamics when the magnetic field is normal to the plane of the discontinuity, but the remaining quantities on both sides of the discontinuity are arbitrary. It is very complicated to obtain the solution of this problem as a limiting case of the problem of the resolution of a discontinuity in an arbitrary field as the tangential components of the magnetic field on both sides of the discontinuity tend to zero. The difficulty of such a limiting process is clearly evident even in the solution of the piston problem in magnetohydrodynamics [14], where for the tangential component of the magnetic field in the undisturbed medium to vanish, three limiting cases are possible, depending upon the value of $P = 4\pi\gamma p/H_x^2$ in the undisturbed medium [15].

The problem of resolution is established in the following form. At the instant $t = 0$ let the thermodynamic parameters of the medium, the velocity, and the normal magnetic field undergo a discontinuity across the plane $x = 0$. Because the conservation laws are not satisfied at such a discontinuity, it will be resolved into some combination of waves. On each side two waves may go out, separated by contact discontinuities. There will be no vorticity discontinuities. For a vorticity discontinuity can exist only behind a shock wave that switches on the field. But this is impossible, because the speed of propagation of such a wave with respect to the gas behind it is just equal to the Alfven speed calculated from the parameters behind the wave.

If it is supposed that every possible wave may exist, then altogether twenty different cases of resolution of a discontinuity are possible, depending upon the properties of the medium on the left and right of the discontinuity. The purpose of the investigation is to determine methods for indicating which of the possible types of resolution exists in each concrete case. The present paper proposes a method of solving this problem, which consists in the construction of a diagram in the plane of $\Delta u = u_0 - u_0'$ and $\Delta v = v_0 - v_0'$ by means of which, knowing Δu and Δv, it is possible to determine the combination of waves by which an initial discontinuity will be resolved.

The medium is assumed ideally conducting. It is also assumed that the solution of the problem under consideration is unique.

Parameters of the medium to the left (right) of the initial discontinuity at $t = 0$ will be denoted by the index $0(0')$. Parameters of the medium to the right of the contact discontinuity for $t > 0$ will be written with a prime.

The symbols S^+, S^-, R^+, R^-, K will indicate respectively the fast and slow shock wave, fast and slow expansion waves, and contact discontinuity. The subscript g indicates that the wave is a gasdynamic one; the index w denotes a shock wave that switches on the tangential component of the magnetic field.

Depending upon the values of the quantities P_0 and P_0' there may be six cases.

1. Consider $P_0 > 1$, $P_0' > 1$. In the Ph plane the components 0, $0'$ correspond to the points P_0, P_0' of the axis of abscissae in Fig. 1.

In [5] it was shown that to combinations consisting of two shock or self-similar waves and a contact discontinuity there corresponds a point in the plane of $\Delta u = u_0 - u_0'$ and $\Delta v = v_0 - v_0'$; to combinations consisting of three shock or self-similar waves and a contact discontinuity there correspond lines in the $\Delta u \Delta v$ plane, these being the boundaries of regions that correspond to combinations consisting of four shock or self-similar waves and a contact discontinuity.

The relations satisfied on a contact discontinuity are [16]

$$[p] = 0, \quad [\mathbf{H}] = 0, \quad [\mathbf{b}] = 0$$

From the first equality it follows that in moving along those lines out of the points P_0, P_0' which correspond to the relationships between P and h appropriate to S^+-, S^--, R^+-, and R^--waves, into which the original discontinuity is resolved, it is possible to arrive at one and the same point in the Ph plane. Henceforth, by studying along which lines out of the points P_0, P_0' it is possible to arrive at the same point in the Ph plane, we find out by which combinations it is possible to resolve the original discontinuity. Then a diagram is constructed in the $\Delta u \Delta v$ plane by means of which, it is possible to determine the combination of waves into which the original discontinuity is resolved. The diagram corresponding to case 1 is constructed in Fig. 2.

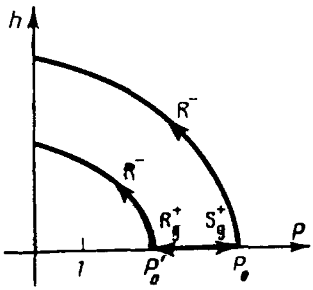

Fig. 1.

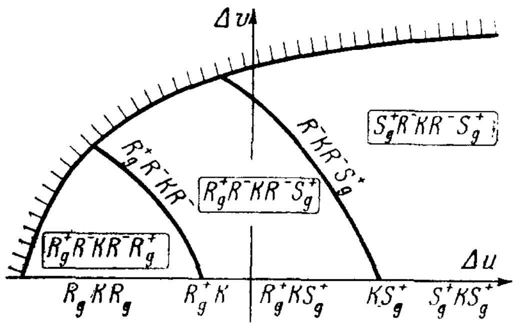

Fig. 2.

In Fig. 1 it is evident that from two shock or self-similar waves and a contact discontinuity it is possible to form R_gKR_g ($R_g^+R_g^-KR_g^-R_g^+$)-, $R_g^+KS_g^+$-, and $S_g^+KS_g^+$-combinations. Furthermore, in the case under consideration the discontinuity may be resolved into one S_g^+-wave going to the right and a contact discontinuity, or into one R_g^+-wave going to the left and a contact discontinuity. All these combinations are represented by corresponding points on the Δu axis in the $\Delta u \Delta v$ plane, because in S_g- and R_g-waves $\Delta v = 0$. Making the assumption of uniqueness of the solution, it is possible to assert that resolution into each of these combinations can be achieved if the Δu and Δv of the discontinuity fall on these points of the $\Delta u \Delta v$ plane.

Henceforth, instead of "the region (line, point) that corresponds to the $R_g^+R^-KR^-S_g^+$ ($R^-KR^-S_g^+$, $R^-KS_g^+$)-combination," we will write "the

$R_g^+R^-KR^-S_g^+$ ($R^-KR^-S_g^+$, $R^-KS_g^+$)-region (line, point)," and so on.

We consider combinations of three shock or self-similar waves and a contact discontinuity. An S_g^+-wave may go to the right (from P_0' to P_0 in Fig. 1), behind which follows an R^--wave that switches on the field; an R^--wave goes to the left, of the same intensity as that to the right. As a result we have the $R^-KR^-S_g^+$-combination. In the *Ph* plane R^--waves are represented by R^--lines going out of the point P_0. This combination of three waves corresponds in the $\Delta u \Delta v$ plane to a line going out of the point that corresponds to the KS_g^+-combination. Moving along this line the intensity of the S_g^+-wave is constant, but the intensity of the R^--wave changes from zero (point KS_g^+) to a maximum (the point of the vacuum line, indicated by cross-hatching in Fig. 2).

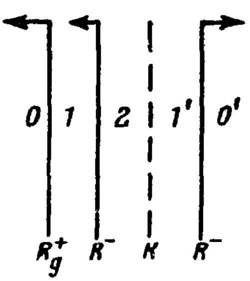

Fig. 3.

Another combination of three shocks: an R_g^+-wave goes to the left (from point P_0 to point P_0', Fig. 1), behind which follows an R^--wave, separated by a contact discontinuity from an R^--wave of the same intensity propagating to the right. This $R_g^+R^-KR^-$-combination corresponds in the $\Delta u \Delta v$ plane to a line going out of the point R_g^+K (at which point the intensity of the R^--wave is equal to zero) and proceeding to the vacuum line.

Points of the vacuum line, and also points lying to the left of the vacuum line, correspond to cases of resolution of an arbitrary discontinuity into a combination of waves including an R^--wave of maximum intensity, after whose passage a vacuum is formed.

The line $R^-KR^-S_g^+$ separates the regions corresponding to the $S^+R^-KR^-S_g^+$- and $R_g^+R^-KR^-S_g^+$-combinations. The line $R^+R^-KR^-$ separates the regions corresponding to the $R_g^+R^-KR^-S_g^+$- and $R_g^+R^-KR^-R_g^+$-combinations.

The figures in the $\Delta u \Delta v$ plane are symmetric with respect to the Δu axis, which follows from the vanishing of the tangential components of the magnetic field ahead of the S_g^+, R_g^+- and R^--waves.

For definiteness we show in the example of the line corresponding to the $R_g^+R^-KR^-$-combination how the equation of the line in the $\Delta u \Delta v$ plane can be written down.

Thus let the resolution be accomplished by the $R_g^+R^-KR^-$-combination. The resolution scheme is depicted in Fig. 3. From Equations (1.4), (1.5), (2.2), and (2.3) of [5] it follows that

$$u_2 = u_1 + \chi_- = u_0 + \chi_+ + \chi_- = u_1' = u_0' - \chi_-'$$

$$v_2 = v_1 \pm \psi_- = v_0 \pm \psi_- = v_1' = v_0' \mp \psi_-'$$

Hence

$$\Delta u = -\chi_+ - \chi_- - \chi_-', \qquad \Delta v = \mp \psi_- \mp \psi_-'$$

These equations describe the line corresponding to the $R^+R^-KR^-$-combination. Moving along this line χ_+ is constant, but the quantities χ_-, χ_-', ψ_- and ψ_-' change from zero to certain maximum values. The equations of the other lines, including the vacuum line, are found analogously. (See also [5] in this connection.)

2. Consider $P_0 > 1$, $P_0' = 1$. In this case the diagram in the $\Delta u \Delta v$ plane is easily obtained from Fig. 2.

The $R^+_g R^- KR^-$-line disappears (it coincides with a portion of the Δu axis), because an R^--wave propagating into gas where $P \leqslant 1$ and $h = 0$ degenerates into an R^-_g-wave, which corresponds in the Ph plane [15] to the segment of the P axis from $P = 1$ to $P = 0$; in such a wave the quantity Δv is equal to zero. There remains one $R^- KR^-S^+_g$-line, separating the $R^+_g R^- KR^-S^+_g$- and $S^+_g R^- KR^-S^+_g$-regions, and extending from the Δu axis to the vacuum line.

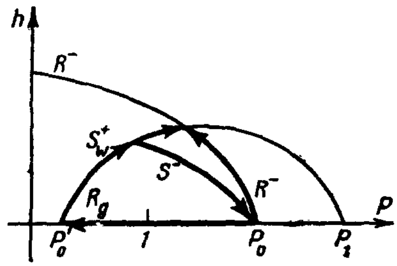

Fig. 4.

3. Consider $P_0 > 1$, $P_0' < 1$ (Figs. 4 and 6 for the Ph plane and Figs. 5 and 7 for the $\Delta u \Delta v$ plane). Now an R^--wave cannot be propagated to the right, because $P_0' < 1$ and $h_0' = 0$, but an S^+_w shock wave can be propagated, which turns on the tangential component of the magnetic field [8, 15, 17]. The relation between P and h in an S^+_w-wave is given by the equation [8]

$$h^2 = 2\eta\left(1 - \frac{\gamma - 1}{2}\eta - P_0\right), \qquad \eta = \frac{P_0}{\gamma - 1}\left\{\left[1 + \frac{2(\gamma-1)}{\gamma}\frac{P - P_0}{P_0^2}\right]^{\frac{1}{2}} - 1\right\}$$

and is depicted in Figs. 4 and 6. Here P_1 is the value of P on the S^+_w-wave for which H_τ behind it is equal to zero.

It can be shown that

$$P_1 = \frac{2\gamma}{\gamma - 1} - \frac{\gamma + 1}{\gamma - 1} P_0$$

An R^--wave can be propagated to the left.

a) Consider for definiteness $P_0 < P_1$. Then from two shock and self-similar waves it is possible to form the $R^- K S_w^+$-, $K S^- S_w^+$-, $S_g^+ K S_g^+$-, $R_g K R_g$-, and $R_g K S_g$-combinations of Fig. 4.

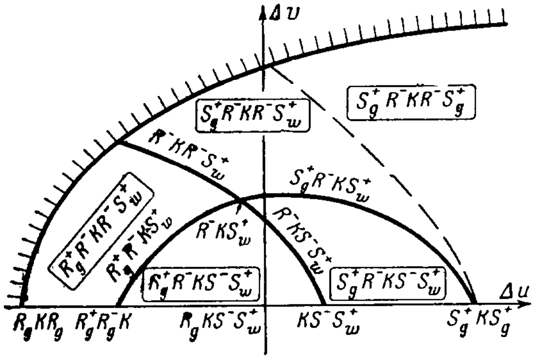

Fig. 5.

Here the symbols S_g and R_g without the + or – sign always indicate gasdynamic shock waves and rarefaction waves which may, generally speaking, be either fast or slow. The last four combinations correspond to portions of the Δu axis. This fact is obvious for the last three combinations. We show that it is true for the $K S^- S_w^+$-combination.

Thus for such a combination [5]

$$\Delta v = \pm \varphi_+' \mp \varphi_-'$$

In this combination the S^--wave has maximum intensity, that is $H_\tau = 0$ behind it; consequently from (2.2) of [5]

$$\varphi_- = h_1' V_1'$$

but behind the S_w^+-wave, switching on the field

$$\varphi_+' = h_1' V_1'$$

Therefore $\Delta v = 0$ for the $K S^- S_w^+$-combination.

From points of the $\Delta u \Delta v$ plane that represent the $R^- K S_w^+$-combination may come lines corresponding to the following combinations, consisting of three shock or self-similar waves and a contact surface (Fig. 5).

1. The $R^- K R^- S_w^+$-combination. The line corresponding to it ends on the vacuum line, where in the Ph plane the R^--line coming out of the point P_0 intersects the h_1 axis.

2. The $R^- K S^- S_w^+$-combination. The line corresponding to it ends on the Δu axis at the point corresponding to the $K S^+ S_w$ ($K S_g$)-combination.

Here the intensity of the R^--wave is equal to zero, and the S^--line beginning at the S_w^+-line goes to the point P_0.

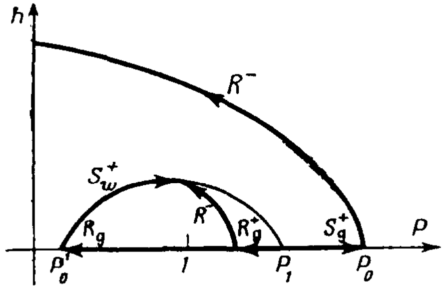

Fig. 6.

3. The $R_g^+R^-KS_w^+$-combination. In the Ph plane the line corresponding to the R_g^+-wave is the section of the straight line coinciding with the P axis to the left from the point P_0 to $P = 1$. At points of this segment R^--lines can begin, going to intersection with the S_w^+-lines. The smaller P is ahead of the R^--wave, the smaller are H_τ and Δv in the wave. An R^--line corresponding to an R^--wave propagating into gas with $P \leqslant 1$ and $h = 0$ degenerates into the portion of the P axis from $P = 1$ to $P = 0$. An R_g^+-wave behind which P becomes less than unity goes continuously into an R_g^--wave [15].

A line corresponding to an $R_g^+R^-KS_w^+$-combination ends at a point lying on the Δu axis that corresponds to an $R_g^+R^-K$-combination.

4. The $S_g^+R^-KS_w^+$-combination. The line corresponding to it ends at a point lying on the Δu axis that corresponds to an $S_g^+KS_g^+$-combination. Moving along this line the intensity of the R^--wave decreases to zero.

We note that to a point of the Δu axis that corresponds to a $KS^-S_w^+$-combination there corresponds also a KS_g-combination, and to the portion of the Δu axis that represents an $S_g^+KS^-S_w^+$-combination there corresponds also an $S_g^+KS_g$-combination.

From the point $S_g^+KS_g^+$ goes out a line (shown dashed in Fig. 5) that separates the $S_g^+R^-KR^-S_w^+$- and $S_g^+R^-KR^-S_g^+$-regions. We note that $P > P_1$ behind the S_g^+-wave moving to the left that enters into the last combination.

In the case under consideration ($H_{\tau 0}' = 0$, $P_0' < 1$), to the left may go a non-evolutionary S_g-wave or an $R^-S_g^+$-combination including a non-evolutionary S_g-wave [18,19] (cf. also [15]), behind which $P < P_1$; this leads to non-uniqueness in the $\Delta u \Delta v$ plane: in this case to one and the same point $\Delta u \Delta v$ may correspond two combinations, for example: $R_g^+R^-KS^-S_w^+$ and $R_g^+R^-KR^-S_g^+$; $S_g^+KS^-S_w^+$ and $S_g^+KS_g$; $KS^-S_w^+$ and KS_g.

However, this indeterminacy is easily eliminated. Thus for arbitrarily small $H_{\tau 0}' \neq 0$ there cannot be an S_g-wave going to the right, and consequently there cannot be a combination containing a non-evolutionary S_g-wave that spoils the determinacy.

b) $P_0 > P_1$ (Fig. 6 for the *Ph* plane, Fig. 7 for the $\Delta u \Delta v$ plane).

In this case there cannot be an $R^- K S_w^+$-combination. From three shock or self-similar wave and a contact discontinuity can be formed the $R_g^+ R^- K S_w^+$- and $R^- K R^- S_g^+$-combinations (Fig. 6). The first combination corresponds in the $\Delta u \Delta v$ plane to a line going from the point $R_g K$ to the point $R_g^+ K S_g^+$ and separating the $R_g^+ R^- K R^- S_w^+$- and $R_g^+ R^- K S^- S_w^+$-regions; the second to a line going from the point $K S^+$ and separating the $R^+ R^- K R^- S_g^+$- and $S_g^+ R^- K R^- S_g^+$-regions.

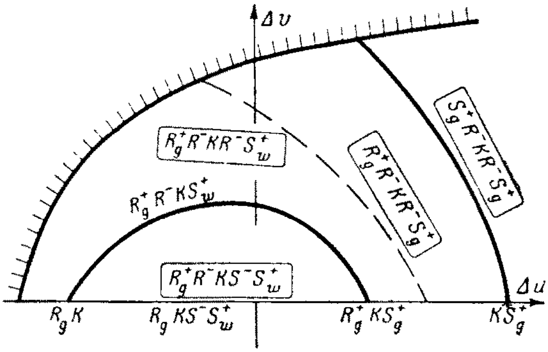

Fig. 7.

In Fig. 7 the line separating the $R_g^+ R^- K R^- S_g^+$- and $R_g^+ R^- K R^- S_w^+$-regions is shown dashed.

On this line the intensity of the S_g^+- and S_w^+-waves is constant, and the intensities of the R^--waves are equal and vary from 0 to a certain finite value.

4. Consider $P_0 < 1$, $P_0' < 1$ and, as before, $P_0 > P_0'$ (Fig. 8 for the *Ph* plane and Fig. 9 for the $\Delta u \Delta v$ plane).

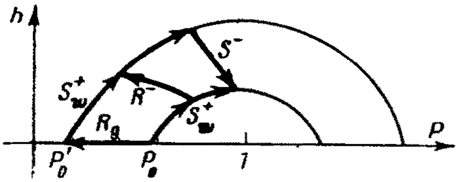

Fig. 8.

We consider the case when the S_w^+-line going out of the point P_0 does not intersect the S_w^+-line going out of point P_0'. The following combinations of three shock or self-similar waves and a contact discontinuity are possible, which correspond to a line in the $\Delta u \Delta v$ plane.

1. The $S_w^+ R^- K S_w^+$-combination. It corresponds to a line beginning at a point on the Δu axis which represents an $R_g^- K$- combination. This line goes over continuously into the line corresponding to the $S_g^+ R^- K S_w^+$- combination. In the *Ph* plane the R^--line, corresponding to the R^--wave

appearing in this combination, does not then end on the S_w^+-line but on the S_g-line. The $S_g^+R^-KS_w^+$-line ends at a point corresponding to the $S_g^+KS_g^+$-combination. The $S_w^+R^-KS_w^+$-line separates the $S_w^+R^-KR^-S_w^+$- and $S_w^+R^-KS^-S_w^+$-regions; the $S_g^+R^-KS_w^+$-line separates the $S_g^+R^-KR^-S_w^+$- and $S_g^+R^-KS^-S_w^+$-regions.

2. The $S_w^+KS^-S_w^+$-combination. It is represented by a line beginning at a point corresponding to the $KS^-S_w^+(KS_g)$-combination, and ending at a point corresponding to the $S_g^+KS^-S_w^+(S_g^+KS_g)$-combination. This line separates the regions corresponding to the $S_w^+R^-KS^-S_w^+$- and $S_w^+S^-KS^-S_w^+$-combinations.

On the line separating the regions that correspond to the $S_w^+R^-KS^-S_w^+$- and $S_g^+R^-KS^-S_w^+$-combinations (shown dashed in Fig. 9), the intensity of the S_g^+-wave going to the right is constant; the intensity of the S_w^+-, R^- and S^--waves varies.

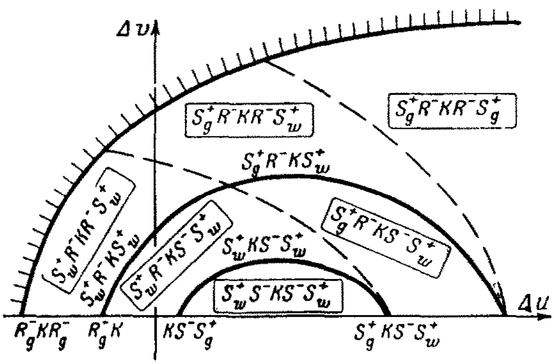

Fig. 9.

On the line separating the $S_w^+R^-KR^-S_w^+$- and $S_g^+R^-KR^-S_w^+$-combinations (also shown dashed in Fig. 9), the intensity of the S_g^+- and S_w^+-waves is constant, whereas the intensity of the R^--wave varies.

In the case discussed in paragraph 4, just as in the case discussed in paragraph 2, combinations involving a non-evoltionary S_g-wave: KS_g etc., are eliminated.

5. Consider $P_0' < 1$, $P_0 = 1$. In this case the diagram in the $\Delta u \Delta v$ plane is easily obtained from Fig. 9. Thus the $S_w^+KS^-S_w^+$-line disappears, because no S_w^+-wave can go out from the point P_0 in the Ph plane. A part of the other line of Fig. 9, which corresponds to the $S_w^+R^-KS_w^+$-combination, also disappears. There remains the $S_g^+R^-KS_w^+$-line, separating the $S_g^+R^-KR^-S_w^+$- and $S_g^+R^-KS^-S_w^+$-regions, and the dashed line separating the $S_g^+R^-KR^-S_w^+$- and $S_g^+R^-KR^-S_g^+$-regions (the vacuum line is definitely present in all cases).

6. Consider $P_0 = 1$, $P_0' = 1$. From two shock or self-similar waves it

is possible to form the $R_g K R_g^-$- and $S_g K S_g$-combinations, where the R_g- and S_g-waves appearing therein have equal intensity. To these combinations correspond points on the Δu axis. Between the Δu axis and the vacuum line is found a region that corresponds to the $S_g^+ R^- K R^- S_g^+$-combination. The S_g^+- and R^--wave appearing therein also have equal intensity.

Thus it is shown that depending upon the values of P_0 and P_0' six different diagrams are possible in the $\Delta u \Delta v$ plane. To solve the problem of the resolution of a discontinuity it is necessary, knowing p_0, p_0', H_x and γ, to form P_0 and P_0' and thus select the type of diagram. To construct the diagram it is also necessary to know ρ_0 and ρ_0'. After this, knowing Δu and Δv at the discontinuity, we ascertain by which combination the initial discontinuity is resolved.

The problem considered in [8] is a special case of the problem investigated in the present work, and is easily obtained from it when

$$P_0 = P_0', \qquad \Delta u = u_0 - u_0' = 0, \qquad \rho_0 = \rho_0'$$

Indeed, it is not difficult to see that with such conditions on the initial parameters the discontinuity may be resolved into either an $S_g^+ R^- K R^- S_g^+$- or an $S_w^+ R^- K R^- S_w^+$-combination.

Diagrams for the case of resolution of a discontinuity when the field is normal to the discontinuity on one side and arbitrary on the other are nearly equivalent to the diagrams obtained by consideration of the general case of resolution [5,6], except that the magnetohydrodynamic waves going to the side of the normal field are to be replaced by gasdynamic waves of the same type.

Moreover, the combination $R_g^+ K R^+$ ($R^+ K R_g^+$) may arise, which is not generally speaking possible in the arbitrary case.

The results obtained have application in astrophysics to the collision of cosmic masses, in magnetohydrodynamics to the investigation of the interaction of magnetohydrodynamic waves, the splitting of non-evolutionary waves, etc.

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